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Publisher's version / Version de l'éditeur:

Canadian Journal of Civil Engineering, 3, 2, pp. 219-228, 1976-06

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BY

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Reprinted from
CANADIAN JOURNAL OF CIVIL ENGINEERING
Vol. 3, No. 2, June 1976
p. 219

Research Paper No. 673
OF THE
DIVISION OF BUILDING RESEARCH

Price 25 cents OTTAWA NRCC 15238

Creep deflection of metal beams in transient heating processes, with particular reference to fire

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Received April 16, 1975

Accepted January 26, 1976

A new numerical technique is described by which the process of creep bending under transient heating conditions can be predicted. It utilizes a convenient creep model, proposed by Dorn and expanded by this author.

The computer simulation of the behavior of three protected steel beams during standard fire tests is discussed. The close agreement between the experimental and computed midspan deflection histories is regarded as the proof for the accuracy of the technique as well as of the creep model employed.

L'article décrit une méthode de calcul numérique nouvelle permettant de prédire l'évolution des déformations flexionnelles dues au fluage dans des conditions thermo-variables. A cette fin, l'auteur exploite un modèle rhéologique commode proposé par Dorn et qu'il a lui-même développé davantage.

Sont discutés les résultats d'une simulation sur ordinateur du comportement de trois poutres d'acier ignifugées supposées soumises à des épreuves d'incendie standard. La concordance serrée observée entre les résultats expérimentaux et les résultats calculés pour l'évolution temporelle de la flèche à mi-travée est considérée comme une démonstration de l'exactitude de la méthode numérique et de la valeur du modèle rhéologique.

[Traduit par la revue]

Can. J. Civ. Eng., 3, 219 (1976)

The long-standing interest in the creep bending of beams is reflected by the large number of papers published on this subject since the early 1930's. A brief review of the various calculation techniques published prior to 1959 was given by Mordfin (1960). Many of these techniques were based on a number of simplifying assumptions concerning the laws of creep and the deformation process, in order to make the problem of creep bending accessible to analytical methods. Further calculation techniques published subsequently by Finnie and Heller (1959), Odqvist and Hult (1962), Anderson *et al.* (1962), Frederick and Lewis (1963), Meck (1965), Goodey (1968), and many others also followed similar approaches.

Popov (1949) described a numerical technique that offered much greater flexibility than any of the analytical methods. This "elastic follow-up" technique was later used by Mordfin (1960) and Mordfin and Halsey (1960) with apparent success.

A weak point in Popov's technique is the assumption that the changes in the fiber stresses are effected by elastic strain increments. This assumption was not used in another numerical technique developed 10 years later by Mendelson *et al.* (1959).

All of these methods are only applicable to isothermal creep processes. Fortunately, most creep problems that occur in practice are such that the temperature and the load can be regarded as reasonably constant throughout the process. Non-isothermal conditions may also occur, but these are usually characterized by either some steady-state temperature distribution in the object or by small temperature fluctuations, and usually do not present serious design problems.

This paper is primarily concerned with those mainly accidental transient heating conditions which may occur as a result of fire or some faulty operation of a device, and which may result in permanent damage or complete destruction of the flexural member in a relatively short time. In such 'runaway' processes not only the temperature is subject to substantial variations, but the load may also undergo certain changes. None of the above-mentioned calculation techniques is applicable to such complex conditions.

Fire Endurance of Beams

In the author's laboratory the need for studying the creep deformation of beams has arisen in connection with calculations concerning the fire endurance[1] of steel-supported floor constructions. Although, for convenience, discussion in this paper will be restricted to fire endurance problems, the calculation method to be introduced may have a much wider area of application.

As pointed out by Harmathy (1967*b*) and Harmathy and Stanzak (1972), irrespective of whether a steel beam in a building is bare or protected against fire exposure either by individual insulation or by ceiling membrane, the procedure for evaluating its performance in fire is basically the same. First, the engineer must have a fairly reliable estimate of the temperature history of the beam during the fire exposure, in other words, he must know the variation of its temperature at some significant locations throughout the process. This information can often be derived by heat flow studies. Unfortunately, the boundary conditions applicable to problems connected with fire (and other 'runaway' processes) are strongly variable.

For standard fire tests, the boundary conditions are defined by American Society for Testing and Materials (ASTM) E-119. For actual fire exposures they can be estimated from the amount of combustible materials present and from the ventilation and certain geometric characteristics of the compartment in which the beam is located, as described by Harmathy (1972).

In the case of beams that are protected against fire, the temperature history is obtained by analyzing the heat flow through the beam insulation, by using numerical techniques, for example, those described by Harmathy (1970) or by Wade and Krokosky (1972). Because these calculations are sometimes rather involved, it is often desirable to find out the temperature history of the beam by subjecting a short but representative section of the protected beam to a rather inexpensive 'small scale' furnace test, in which the temperature of the environment is faithfully reproduced. Such small scale testing facilities are available at the Division of Building Research, National Research Council of Canada, and at several fire testing organizations in North America. The Canadian facilities have been described by Blanchard and Harmathy (1964).

In the following discussion it will be taken for granted that the temperature history of the beam is known. The task is now to calculate either the permanent damage that the beam would suffer after a certain fire exposure, or the time that it would fail if the fire is not extinguished. This latter information is obviously of great importance for engineers concerned with the fire protection of buildings.

[1]Fire endurance is the time for which a building element can withstand fire and give protection from it during an actual fire or a standard fire test performed according to ASTM E-119.

Introductory Remarks

Because of the transient nature of the heating, fixed and/or restrained beam ends would induce time-dependent axial forces and moments and thus substantially increase the complexity of the deformation process. To avoid these difficulties, the scope of this paper is limited to simply supported beams; the effect of fixed or restrained ends will be discussed in a future paper.

Even when the beam ends are simply supported, the problem is still complex enough to necessitate some degree of idealization of the process and the material. Some readers will undoubtedly be interested in the nature of these idealizations in order to understand the limitations of the numerical technique to be described. The practicing engineer, on the other hand, probably prefers to skip over these details and concentrate more on the practical application of the technique. Keeping in mind these two types of readership, the material is arranged in such a way as to satisfy the practicing engineer first and discuss the missing logical details in the last two sections of the paper.

Preparation of Problem for Numerical Analysis

An important feature of the calculation technique is that the deflection history of the beam during the transient heating process is determined from the analysis of its stress–strain history at a single cross section, namely that at the mid-span.

As is usual in numerical techniques, the variables involved in the process are defined only on a lattice of points within the regions of interest. The space lattice consists of a number of points coincident with the centroids of

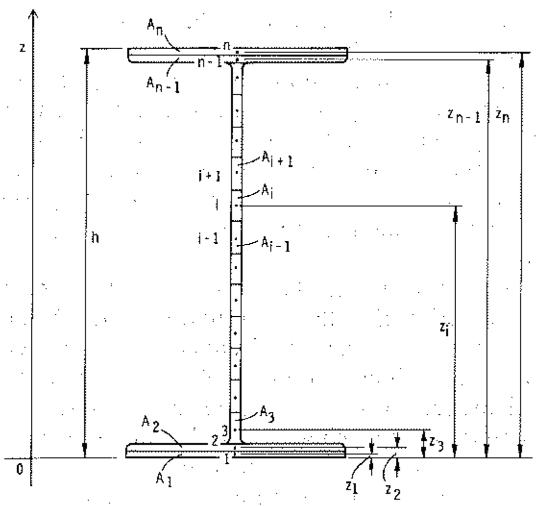

FIG. 1. Subdivision of a beam.

contiguous elementary volumes of the beam. These elementary volumes are obtained by subdividing the beam by planes parallel to the plane of bending. The base areas of the volume elements lie on the mid-span cross section. In reference condition (*i.e.* in stress free condition and at a selected reference temperature) they extend a unit length along the fibers.

Figure 1 shows a typical way of performing the subdivision of a wide-flange beam. The centroids of the elements are numbered[2] 1, 2, . . . i, . . . n. Their distances from the bottom flange of the beam, selected as the origin for the z axis, are denoted by z_1, z_2, . . . z_i, . . . z_n.

The time coordinate for the process is also subdivided into a multitude of elementary periods. The lattice points along the time axis will be denoted by t^0, t^1, t^2, . . . t^j, . . . , and the elementary periods, namely the periods $t^0 < t \leqslant t^1$, $t^1 < t \leqslant t^2$, . . . $t^{j-1} < t \leqslant t^j$, . . . , by Δt^1, Δt^2, . . . Δt^j, . . . , respectively.

As will be pointed out later, unlike the space lattice points, the time lattice points usually cannot be selected in advance for the entire duration of the process.

Input Information

In addition to details of the subdivision of beam, namely A_i and z_i ($i = 1, 2, \ldots n$), various other data are also needed for defining the problem. A complete set of all these data forms the input information. It consists of data connected with (i) the temperature history of the beam, (ii) its material properties, and (iii) the load it carries.

Since, as mentioned earlier, the deflection history of the beam will be calculated from the stress–strain history of the mid-span cross section, it is sufficient to know the variation of the temperature in this cross section only.

If the temperature history is obtained from a simulated fire exposure, as described earlier, it may be prohibitive to install thermocouples at each point of the beam selected as a lattice point in the calculations. Experience has shown that it is sufficient to measure the temperature only at three points: on the lower flange, at the web center, and on the upper flange, and to derive the temperature at the lattice points from the assumption that the variation of temperature along the depth of the beam can be described by a quadratic equation.

Figure 2 shows the variation of temperature of the lower flange, $T_L(t)$, of the web center, $T_W(t)$, and of the upper flange, $T_U(t)$ during three fire tests reported by Stanzak and Harmathy (1968). With information of this sort and with the aid of the said assumption, the temperature of the beam at all of the selected lattice points and at any $t = t^j$ time level can be expressed as follows:

$$[1] \quad T_i^j = T_L^j + (-3T_L^j + 4T_W^j - T_U^j)\frac{z_i}{h} + (2T_L^j - 4T_W^j + 2T_U^j)\left(\frac{z_i}{h}\right)^2$$

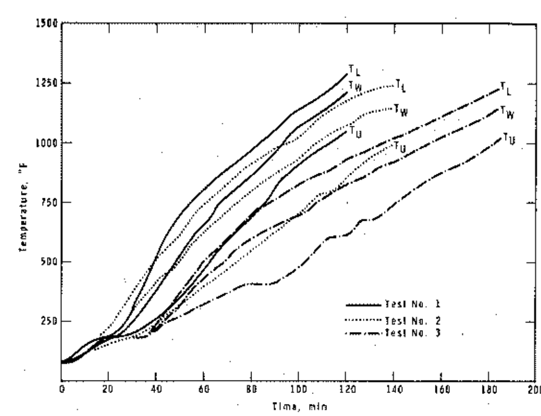

FIG. 2. Variation of beam temperature in three simulated fire exposures.

[2]A summary of the notation is given at the end of this paper.

From among the many material properties the following ones are needed in the computations: the modulus of elasticity, E, and thermal expansion, η, both as functions of the temperature, the activation energy for creep, ΔH, and finally, two creep parameters, namely the Zener–Hollomon parameter, Z, and a yet unnamed parameter, ϵ_I, the last two as functions of the stress. These material properties are generally presented in the form of empirical equations. To an ASTM E-36 steel the following empirical equations apply:

[2] $E(T) = 30 \times 10^6 - 9.3(T - 460)^2$

[3] $\eta(T) = 6.21 \times 10^{-6}(T\text{-}535) + 0.1636 \times 10^{-8}(T\text{-}535)^2$

[4] $\Delta H/R = 70\,000$

[5] $\epsilon_I(\sigma) = 1.7 \times 10^{-10}|\sigma|^{1.75} \operatorname{sgn} \sigma$

[6a] $Z(\sigma) = 0.0261|\sigma|^{4.7} \operatorname{sgn} \sigma$ if $|\sigma| \leqslant 15\,000$

[6b] $Z(\sigma) = 1.23 \times 10^{16} e^{0.0003|\sigma|} \operatorname{sgn} \sigma$ if $|\sigma| > 15\,000$

[The information presented in [4] to [6b] was derived by Harmathy (1967a, b) and Harmathy and Stanzak (1970).]

In agreement with convention, compressive stresses and strains indicating shortening of fibers are regarded as negative quantities.

The load conditions will be characterized by the bending moment at the mid-span at the beginning of the process ($t = t^0$), M^0.

Calculation Procedure

When removed from the reference condition, the total unit deformation (total strain) of the ith elementary volume at some $t = t^j$, namely $(\epsilon_t)_i^j$, will consist of three contributions:

[7] $$(\epsilon_t)_i^j = \frac{\sigma_i^j}{E(T_i^j)} + \eta(T_i^j) + (\epsilon_c)_i^j$$

The first term on the right side of [7] can be recognized as the elastic strain. The second term is the thermal strain brought about by a change in the temperature. These two types of strains are quasi-instantaneous and fully recoverable responses of the material to loading and to any departure from the reference temperature, respectively.

The third term represents the 'creep' strain which, according to the creep model used in this paper, is interpreted as approximately representing all time-dependent and non-recoverable strains, and depends strongly on both the loading conditions and the temperature.

It is because of the presence of this creep deformation that the deformation of the beam at some $t = t^j$ time level cannot be calculated from the initial conditions alone and that the calculation must be based on the stress–strain conditions prevailing a very short time earlier, namely at the $t = t^{j-1}$ time level.

To find out the deformation of the beam at some $t = t^j$ time level, one must know, therefore, in addition to the input information, the distribution of stress, creep strain, and 'compounded' creep strain[3] at the $t = t^{j-1}$ time level, *i.e.* the values of σ_i^{j-1}, $(\epsilon_c)_i^{j-1}$, and $(\bar{\epsilon}_c)_i^{j-1}$, respectively ($i = 1, 2, \ldots n$).

The calculation procedure is now as follows: Calculate first the values of two parameters, a^j and b^j:

[8] $$a^j = \frac{u^j w^j - v^j W^j}{(u^j)^2 - U^j v^j}$$

[9] $$b^j = \frac{u^j W^j - U^j w^j}{(u^j)^2 - U^j v^j}$$

In these equations u^j, U^j, . . . etc. are symbols for the following expressions:

[10] $$u^j = \sum_{i=1}^{n} z_i A_i E(T_i^j)$$

[11] $$U^j = \sum_{i=1}^{n} (z_i)^2 A_i E(T_i^j)$$

[12] $$v^j = \sum_{i=1}^{n} A_i E(T_i^j)$$

[13] $$w^j = \sum_{i=1}^{n} A_i E(T_i^j)\{\eta(T_i^j) + (\epsilon_c)_i^{j-1} + (\Delta\epsilon_c)_i^j\}$$

[14] $$W^j = \sum_{i=1}^{n} z_i A_i E(T_i^j)\{\eta(T_i^j) + (\epsilon_c)_i^{j-1} + (\Delta\epsilon_c)_i^j\} - M^j$$

[3]For the interpretation of the 'compounded' creep strain see [18].

In [13] and [14] $(\Delta\epsilon)_i^j$ represents the creep strain increment during the period $t^{j-1} < t \leqslant t^j$, and can be expressed as

[15] $$(\Delta\epsilon_c)_i^j = Z(\sigma_i^{j-1}) \coth^2\left(\frac{(\bar{\epsilon}_c)_i^{j-1}}{\epsilon_l(\sigma_i^{j-1})}\right) \times \left\{\exp\left(-\frac{\Delta H}{R}\frac{2}{T_i^{j-1} + T_i^j}\right)\right\} \Delta t^j$$

and in [14]

[16] $$M^j = M^0\, e^{-ry^{j-1}}$$

The last equation expresses, in an empirical form, the fact that, in processes connected with fire, the bending moment decreases with increasing deflection owing to the increasing participation of the supported deck in transferring the load directly to the walls and columns of the building. r is the 'load resistance' of the deck and can be calculated or determined from simple experiments, as described by Harmathy (1967*b*).

In [13] and [14], $(\epsilon_c)_i^{j-1}$ is the actual creep strain (at the time level $t = t^{j-1}$) formed as the sum of the 'initial' creep strain, $(\epsilon_c)_i^0$, and the creep strain increments, $(\Delta\epsilon_c)_i$'s during the $\Delta t, \Delta t^2, \ldots \Delta t^{j-1}$ elementary periods:

[17] $$(\epsilon_c)_i^{j-1} = (\epsilon_c)_i^0 + (\Delta\epsilon_c)_i^1 + (\Delta\epsilon_c)_i^2 + \ldots + (\Delta\epsilon_c)_i^{j-1}$$

In [15], on the other hand, $(\bar{\epsilon}_c)_i^{j-1}$ is the 'compounded' creep strain (at $t = t^{j-1}$) formed as the sum of the absolute values of the terms in [17].

[18] $$(\bar{\epsilon}_c)_i^{j-1} = |\epsilon_c|_i^0 + |\Delta\epsilon_c|_i^1 + |\Delta\epsilon_c|_i^2 + \ldots + |\Delta\epsilon_c|_i^{j-1}$$

The initial creep strain is a fictitious creep strain introduced merely as a mathematical convenience. The values are defined quite arbitrarily, as follows:

[19*a*] $$(\epsilon_c)_i^0 = \frac{1}{100}\frac{\sigma_i^0}{E(T_i^0)} \quad \text{if} \left|\frac{\sigma_i^0}{E(T_i^0)}\right| > 10^{-6}$$

[19*b*] $$(\epsilon_c)_i^0 = 10^{-8}\,\text{sgn}\,\sigma_i^0 \,\text{if} \left|\frac{\sigma_i^0}{E(T_i^0)}\right| \leqslant 10^{-6}$$

All information is now available for the calculation of the parameters a^j and b^j. The distribution of the total strain and of the stress at the $t = t^j$ time level is then calculated from the following two equations:

[20] $$(\epsilon_t)_i^j = a^j z_i + b^j$$

[21] $$\sigma_i^j = E(T_i^j)\{a^j z_i + b^j - \eta(T_i^j) - (\epsilon_c)_i^{j-1} - (\Delta\epsilon_c)_i^j\}$$

Finally, the central deflection of the beam is obtained as:

[22] $$y^j = \frac{L^2}{\pi^2}\frac{(\epsilon_t)_1^j - (\epsilon_t)_n^j}{z_n - z_1}$$

Starting with the initial values σ_i^0, $(\epsilon_c)_i^0$, and y^0 (those prevailing at $t = t^0 = 0$), the distribution of σ, ϵ_t, and the value of y can thus be determined by step-by-step calculations for each successive $t = t^1, t^2, t^3, \ldots t^j, \ldots$ time level.

The initial conditions are conveniently selected as those that occur on the imposition of load. If, in addition, the reference temperature is taken as the (uniform) temperature of the beam at this moment, in the absence of thermal and creep strains, the values of σ_i^0 and $(\epsilon_t)_i^0$ can be calculated from Hooke's law and the known value of the mid-span moment, M^0. Then y^0 is obtained from [22], and the fictitious initial creep strain, $(\epsilon_c)_i^0$, from [19a] or [19b].

Because of the labour involved in the calculations, the procedure is usually programmed for computer execution. The procedure followed by the author is somewhat more sophisticated than that described earlier. In his program, σ_i^{j-1}, $(\bar{\epsilon}_c)_i^{j-1}$, and y^{j-1}, which appear in [15] and [16], are replaced by the following expressions:

[23] $$\hat{\sigma}_i^j = \sigma_i^{j-1} + \tfrac{1}{2}(\sigma_i^{j-1} - \sigma_i^{j-2})\frac{\Delta t^j}{\Delta t^{j-1}} \quad \text{for} \quad j \geqslant 2$$

[24] $$(\hat{\bar{\epsilon}}_c)_i^j = (\bar{\epsilon}_c)_i^{j-1} + \tfrac{1}{2}((\bar{\epsilon}_c)_i^{j-1} - (\bar{\epsilon}_c)_i^{j-2})\frac{\Delta t^j}{\Delta t^{j-1}} \quad \text{for} \quad j \geqslant 2$$

[25] $$\hat{y}^j = y^{j-1} + \tfrac{1}{2}(y^{j-1} - y^{j-2})\frac{\Delta t^j}{\Delta t^{j-1}} \quad \text{for} \quad j \geqslant 2$$

respectively, in other words, by the estimated average values of these variables obtained by linear extrapolation from their values at the $t = t^{j-1}$ and $t = t^{j-2}$ time levels. Since the variation of these variables may be quite substantial at advanced stages of the deformation process, even during a relatively short $t^{j-1} < t \leqslant t^{j}$ elementary period, using these extrapolated values will improve the accuracy of the numerical procedure without significantly adding to the complexity of the calculations.

The criteria for the stability and convergence of the numerical procedure are not known. With the use of a computer, however, this fact presents no problem. The accuracy of the results can be checked by repeating the computation two or three times, with gradually reduced elementary time periods.

In the case of structural steel, up to about 1210 R (750 °F), the deformation process is generally controlled by the quasi-instantaneous responses of the material to the load and to changes in temperature; consequently, large time increments (0.1 h or more) can be tolerated. The creep deformation may not become significant until some parts of the beam reach 1360 R (900 °F). Above 1460 R (1000 °F) the creep is normally the controlling factor and, therefore, the time increments must be substantially reduced. In the computations performed by the author, the time increments were reduced to 0.005 h as soon as the highest temperature in the beam exceeded 1460 R.

Experimental Verification

To determine the accuracy of the calculation procedure, computer studies have been performed to simulate three fire tests the results of which have been reported earlier by Stanzak and Harmathy (1968). The tests were performed on protected steel beams made of ASTM A-36 steel and weighing 1.417 lb/in. (25.5 kg/m). Figure 1 depicts the cross section of the beams, and the subdivision of the cross section into 16 elementary areas.

The following additional information applies: span $L = 186$ in. (472.4 cm); depth $h = 8$ in. (20.3 cm); mid-span moment $M^0 = 205000$ lb-in. (23160 Nm); and load-resistance of deck $r = 0.05$ in.$^{-1}$ (1.3 mm^{-1}). The beams carried a simulated deck section. The load-resistance of the decks was estimated as described by Harmathy (1967*b*).

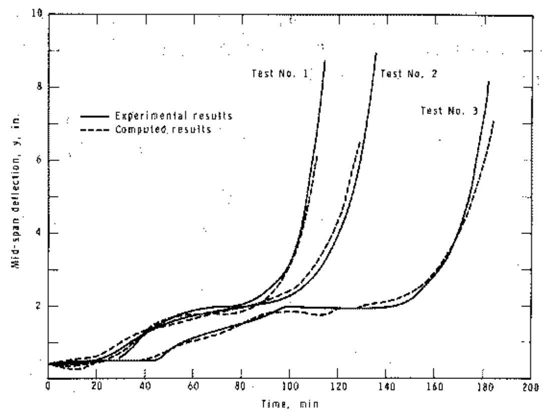

FIG. 3. Comparison of measured and calculated mid-span deflections.

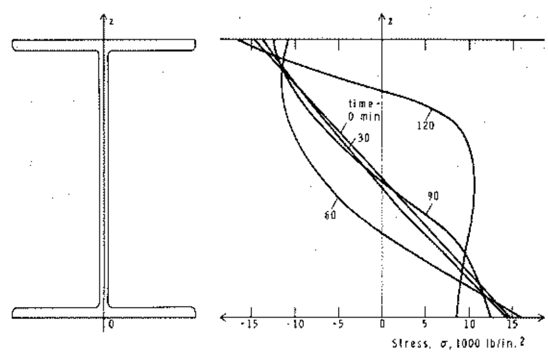

FIG. 4. Computed variation of stress in beam at mid-span during test No. 2.

The mid-span temperature of the beams was measured at three locations: at the lower flange, at the mid-height of the web, and at the upper flange. The $T_L(t)$, $T_W(t)$, and $T_U(t)$ curves in Fig. 2 relate to these three fire tests.

In Fig. 3 the measured mid-span deflections are compared with those obtained by the use of a computer. The fast rates of deflection toward the end of the tests indicate imminent failure. The close agreement between the experimental and computed mid-span deflection histories is regarded as the proof for the accuracy of the calculation technique.

Figure 4 shows the computed variations of the stress distribution at the mid-span during test No. 2 and the significant shift of the neutral plane, brought about partly by thermal stresses and partly by creep. The shift greatly accelerates at higher temperatures where creep becomes the prime mechanism of deformation.

Modeling the Beam and the Process

The scrutinizing reader has, no doubt, already recognized a few implicit assumptions on which the numerical procedure has been built. Indeed, four fundamental assumptions were used, two of which are common with those employed in the elementary theory of elastic bending. These are:

1. The beam is long in comparison with its depth and, therefore, the deformations resulting from transverse shear stresses are negligible. The cross section is symmetrical about the plane of bending, and the beam is loaded in such a way that twisting does not occur.
2. Plane sections normal to the axis of the beam remain plane during the deformation process. This so-called Bernoulli's hypothesis was experimentally verified for creep by McCullough (1933).

To reduce the labour involved in the calculations, two further assumptions have been employed.

3. The temperature, at any given time, varies only along the depth of the beam.
4. The deflected shape of the beam can be described by a sine curve.

Assumption 3 is normally a fair approximation of the actual conditions. Assumption 4 is reasonably accurate for beams under uniform loads and, to a lesser degree, for beams loaded with concentrated forces symmetrically distributed about the mid-span. In the latter cases, the calculations can be based on the assumption of an equivalent uniform load, defined as that which produces the same maximum bending moment as the actual load.

Obviously, assumptions 1 and 3 were tacitly utilized in subdividing the beam into elementary volumes by planes parallel to the plane of bending, and assumption 3 made it possible to describe the temperature distribution by [1].

The reader can also recognize now that it is assumption 4 which formed the basis for [22], and for the simple concept of expressing the deflection history of the beam from the analysis of its stress–strain history at the mid-span alone. (The procedure to be followed if assumption 4 is not applicable was described earlier by Harmathy (1967*b*).)

Assumptions 1 and 2 will now permit the author to present the background information for [8] to [14].

Since the beam is simply supported and unrestrained, it carries no axial load, and therefore, the following equation will express the condition of equilibrium of internal forces at the mid-span at some $t = t^j$ time level.

$$[26] \qquad \sum_{i=1}^{n} \sigma_i^j A_i = 0$$

Furthermore, since the moment of stresses in the mid-span cross section must be equal to the instantaneous mid-span moment, M^j, resulting from the applied load,

$$[27] \qquad \sum_{i=1}^{n} \sigma_i^j z_i A_i = -M^j$$

Finally, Bernoulli's hypothesis implies a linear variation of the total strain along the z axis, which was earlier expressed by [20].

By combining [7] (with $(\epsilon_c)_i^j$ expressed as its value at $t = t^{j-1}$ plus the creep strain increment during the $t^{j-1} < t \leqslant t^j$ elementary period) and [20] the following equation is obtained:

$$[28] \qquad a^j z_i + b^j = \frac{\sigma_i^j}{E(T_i^j)} + \eta(T_i^j) + (\epsilon_c)_i^{j-1} + (\Delta\epsilon_c)_i^j$$

Equation [21] is the result of expressing σ_i^j from the above equation. By substituting this expression of σ_i^j into [27] and [26], one obtains:

$$[29] \qquad \sum_{i=1}^{n} z_i A_i E(T_i^j)\{a^j z_i + b^j - \eta(T_i^j) - (\epsilon_c)_i^{j-1} - (\Delta\epsilon_c)_i^j\} = -M^j$$

$$[30] \qquad \sum_{i=1}^{n} A_i E(T_i^j)\{a^j z_i + b^j - \eta(T_i^j) - (\epsilon_c)_i^{j-1} - (\Delta\epsilon_c)_i^j\} = 0$$

Finally, by expressing a^j and b^j from these two equations, [8] to [14] result.

Modeling the Material

As has been shown in Fig. 4, when bending with creep occurs, the stresses in various fibres of the beam are liable to change even if the temperature and load remain constant. Since the information concerning the creep characteristics of materials are generally derived from tests performed at constant temperatures and constant stresses, assuming that this information is applicable to creep processes occurring at variable temperature and stress is a prerequisite to an attempt to formulate the problem. Such an assumption is, in fact, equivalent to assuming the existence of a mechanical equation of state which is capable of describing uniquely the instantaneous rheological state of the object, irrespective of its past stress and temperature history. Although such equations of state do not seem to exist, it is believed that if the stresses in the various fibres do not change very fast, such an assumption will not result in serious errors.

In describing bending processes at constant temperature, it is common to use as the equation of state either the time-hardening or the strain-hardening laws (Odqvist and Hult 1962; Marriott and Leckie 1963). Neither these nor other creep laws are applicable, however, to creep bending at transient heating conditions.

Harmathy (1967*a*, *b*) showed that the following equation can be regarded as a comprehensive equation of state, and yields a satisfactory accuracy in describing the time-dependent deformation (creep[4]) of polycrystalline materials under a great variety of temperature and stress conditions:

[31] $$\frac{1}{Z}\frac{d\epsilon_c}{d\theta} = \coth^2\left(\frac{\epsilon_c}{\epsilon_I}\right)$$

In this equation ϵ_c is the time-dependent (creep) strain and θ is a "temperature compensated time" defined by Dorn (1956) as:

[32] $$\theta = \int_0^t e^{-\Delta H/RT} dt$$

where ΔH is the activation energy of creep, R is the gas constant, and T is the absolute temperature.

ϵ_I and Z are creep parameters dependent solely on the applied stress:

[33] $$\epsilon_I = \epsilon_I(\sigma)$$

[34] $$Z = Z(\sigma)$$

The second of these is the well-known Zener–Hollomon parameter (Zener and Hollomon 1946).

The two creep parameters and the activation energy of creep, ΔH, were studied by Harmathy (1967*a*, *b*) and Harmathy and Stanzak (1970) for three steels used in North America, and by Thor (1972) for a number of steels used in Europe. Studies by Harmathy (1967*a*, *b*) and Thor (1971, 1972*a*, *b*) indicated very good agreement between information developed from [31] and from experiments performed at variable temperatures.

It has been assumed in this paper that the creep model is applicable to both tensile and compressive deformations.

Equation [15] can be recognized as a finite-difference form, for some $t^{j-1} < t \leqslant t^j$ period, of [31], with a slight modification, namely that on the right side of the equation $(\epsilon_c)_i^{j-1}$ is replaced by $(\bar{\epsilon}_c)_i^{j-1}$. The explanation for using here the 'compounded' creep strain instead of the ordinary creep strain is as follows.

Since, in a process of bending with creep, the position of the neutral plane shifts considerably along the z axis, even if the load remains constant, certain fibers that were previously subjected to compression may come under tension, or vice versa. In this process some $(\epsilon_c)_i$'s will also change signs.[5] The time at which this occurs in a particular fiber does not usually coincide with the time at which the stress changes signs in the same fiber. Thus the argument of the hyperbolic cotangent term may become zero for some fibers at some stage or other of the deformation process. This term is, however, not defined for zero argument.

The practice of using the 'compounded' creep strain, as defined by [18], eliminates the mathematical problem and, at the same time, can also be defended from the point of view of the dislocation theory. Discussion on this aspect of the problem is, however, beyond the scope of the present paper.

Even with the new variable, $\bar{\epsilon}_c$, the argument of the hyperbolic cotangent term is still zero at the beginning of the creep process. To circumvent this problem, a small initial value $(\epsilon_c)_i^0$ is added to the creep strain. Its values are defined by [19a] and [19b].

[4]For reasons discussed in the said paper, all time dependent deformation will be referred to as creep and will be regarded as essentially non-recoverable. Equation [31] is not applicable to the tertiary period of creep.

[5]Equation [31] was derived from conventional creep tests with $\sigma \simeq$ const. In the course of such tests the above conditions cannot occur. Thus, [31] remains valid in its original form for correlating creep test results, or for deformation processes in which the stresses do not change signs.

Conclusion

The creep deflection of beams in transient heating processes is of considerable interest to engineers working in the field of fire protection and possibly to some others concerned with mainly accidental 'runaway' processes. The analytical and numerical methods published prior to 1967 are not adaptable to the calculation of beam deformation under transient conditions.

A numerical technique has been described in this paper. The technique utilized a creep model originally suggested by Dorn (1956) and expanded by the author to suit certain practical requirements. Although the technique is applicable to any load pattern, the detailed discussion in this paper has been limited to load distributions which result in quasi-sinusoidal beam deflection. The technique is utilized to simulate the process of deformation of protected steel beams during fire tests.

Acknowledgment

This paper is a contribution from the Division of Building Research, National Research Council of Canada and is published with the approval of the Director of the Division.

Notation

a = parameter, in.$^{-1}$
A = elementary area, in.^{2}
b = parameter, dimensionless
E = modulus of elasticity, lb/in.^{2}
h = depth of beam, in.
ΔH = activation energy of creep, Btu/lb mole
i = $=1, 2, 3, \ldots n$
L = span, in.
M = mid-span moment resulting from applied load, lb-in.
n = number of elements in spatial subdivision
r = load resistance of deck, in.$^{-1}$
R = gas constant, Btu/lb mole R
t = time, h
T = temperature, R
u = variable, defined by [10], lb-in.
U = variable, defined by [11], lb-in.
v = variable, defined by [12], lb
w = variable, defined by [13], lb
W = variable, defined by [14], lb-in.
y = mid-span deflection, in.
$\hat{y}$ = average value for y for $t^{j-1} < t \leqslant t^j$, in.
z = dimension along depth of beam, in.
Z = Zener–Hollomon parameter, h^{-1}

Greek Letters

Δ = increment
ϵ = strain, in./in.
$\bar{\epsilon}$ = 'compounded' creep strain, in./in.
$\hat{\epsilon}$ = average value for $\bar{\epsilon}$ for $t^{j-1} < t \leqslant t^j$, in./in.
ϵ_1 = creep parameter, in./in.
η = strain resulting from temperature changes, in./in.
θ = temperature compensated time, h
σ = stress, lb/in.^{2}
$\hat{\sigma}$ = average value for σ for $t^{j-1} < t \leqslant t^j$, lb/in.^{2}

Subscripts

c = creep (or time-dependent)
i = pertaining to the i th elementary area or volume
L = of the lower flange
t = total
U = of the upper flange
W = of the web center

Superscripts

0, 1, 2, . . . $(j-1)$, j pertaining to the 0 th, 1st, 2nd, . . . $(j-1)$ th, j th time level, or to elementary periods preceding these time levels.

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