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# CORRELATION OF REVERBERANT SOUND FIELDS USING A $k$ -TRANSFORM TECHNIQUE

by

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# LA CORRELATION DES CHAMPS SONORES REVERBERANTS PAR UNE METHODE DE "K" TRANSFORMES

## *SOMMAIRE*

L'auteur décrit une méthode de transformées servant à calculer la corrélation spatiale de divers genres de champs acoustiques. On insiste sur le fait que l'emploi d'une méthode de "k" nombres pour déterminer un champ sonore équivaut à la synthèse d'un champ sonore en tant que sommation d'une série d'ondes planes.

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## Correlation of Reverberant Sound Fields Using a $k$ -Transform Technique

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A transform method is described for calculating the spatial correlation for various kinds of acoustic fields. The method emphasizes the equivalence between using a  $k$ -number approach for specifying a sound field and the synthesis of a sound field as a summation of a series of plane waves.

The usual way of determining the spatial correlation of a three- or two-dimensional reverberant sound field is to decompose the original field into a system of plane waves, calculate the effect for a small bundle of such waves, and then integrate for the whole set.<sup>1</sup>

Specifically, in the three-dimensional case, the correlation function  $\rho(d, \tau)$  for spatial separation  $d$  and time difference  $\tau$  is given by

$$\rho(d, \tau) = (4\pi)^{-1} \int_0^{2\pi} \int_0^\pi \rho\left(\tau + \frac{d \cos\theta}{c}\right) \sin\theta d\theta d\varphi, \quad (1)$$

where  $c$  is the propagation velocity, and  $\theta, \varphi$  are the polar and azimuth angles in spherical coordinates; the factor  $4\pi$  ensures normalization. When we are considering sine waves we may write

$$\rho\left(\tau + \frac{d \cos\theta}{c}\right) = \cos\omega\left(\tau + \frac{d \cos\theta}{c}\right), \quad (2)$$

and we may write Eq. 1 for  $\tau=0$  as

$$\begin{aligned} \rho(d, 0) &= (4\pi)^{-1} \left| \frac{\sin(\omega d \cos\theta/c)}{\omega d/c} \right|_0^\pi 2\pi \\ &= \frac{\sin(\omega d/c)}{\omega d/c}. \end{aligned} \quad (3)$$

### I. TRANSFORM METHOD

Sometimes when dealing with sound transmission problems it is more convenient to deal with the wave-number parameter  $k$  than the angle of incidence  $\theta$ . The following examples emphasize the equivalence of the two approaches. The first example evaluates the three-dimensional reverberant field case given above.

#### A. Three-Dimensional Reverberant Field

Let  $p_\Omega(\Omega) \cdot d\Omega$  represent the distribution of energy within solid angles  $\Omega$  and  $\Omega + d\Omega$ . For an azimuthally sym-

metric system, we may write the distribution as  $p_\theta(\theta)$  where, of course,

$$p_\Omega(\Omega) d\Omega = p_\theta(\theta) d\theta.$$

If we consider a three-dimensional reverberant space, then

$$p_\theta(\theta) = (4\pi)^{-1} 2\pi \sin\theta = \frac{1}{2} \sin\theta. \quad (4)$$

For such an axially symmetric system we may write

$$k f_k(k) dk = 2\left(\frac{1}{2} \sin\theta d\theta\right), \quad (5)$$

where we have now expressed the distribution in terms of  $f_k(k)$ . Equation 5 equates the energy for a band  $dk$  in the  $k$  plane to that in band  $d\theta$  in the  $\theta$  plane.

Writing  $k = k_\alpha \sin\theta$ , where  $k_\alpha$  is the wave propagation number, then

$$k f_k(k) = \sin\theta \frac{d\theta}{dk} = \frac{k}{k_\alpha^2} \left(1 - \frac{k^2}{k_\alpha^2}\right)^{-\frac{1}{2}}. \quad (6)$$

Equation 6 is a measure of the power  $k$  spectrum of the reverberant sound field. By taking the inverse transform—in this case Hankel transform as we have assumed axial symmetry—and using the Wiener-Khinchine theorem, we may derive the correlation function between two points separated in space. Thus we may write for the spatial correlation coefficient

$$\int_0^{k_\alpha} \frac{k}{k_\alpha^2} J_0(kr) \left(1 - \frac{k^2}{k_\alpha^2}\right)^{-\frac{1}{2}} dk = \frac{\sin k_\alpha r}{k_\alpha r}. \quad (7)$$

The upper limit of  $k$  must be  $k_\alpha$  to correspond to the upper limit of  $\theta = \pi/2$ .

An interesting sidelight is obtained from Eq. 7. The right-hand side is such that one would expect a constant bounded  $k$  spectrum if the correct variables were chosen. If one chooses  $k_z$ , the wavenumber perpendicular to the surface being considered, then

$$f(k_z) dk_z = 2\pi \sin\theta d\theta$$

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and

$$f(k_z) = \text{constant}(k_\alpha)^{-1}.$$

Then by a suitable choice of axes we may simplify the derivation of Eq. 7.

### B. Incident Field from an Annulus

For this case, we shall take  $p_\Omega(\Omega) = \delta(\Omega - \Omega_0)$ ; i.e., the sound field is restricted to a narrow band of solid angles around  $\Omega_0$ , where  $\delta(\Omega - \Omega_0)$  is the impulse function and

$$\int_{-\infty}^{\infty} \delta(\Omega - \Omega_0) d\Omega = 1.$$

We may then write

$$p_\theta(\theta) = 2\pi \sin\theta \delta(\Omega - \Omega_0). \quad (8)$$

Proceeding as before,

$$k f_k(k) = 2\pi (k/k_\alpha^2) \delta(\Omega - \Omega_0) (1 - k^2/k_\alpha^2)^{-\frac{1}{2}}. \quad (9)$$

Similarly, the inverse transform will be given by

$$\int_0^{k_\alpha} 2\pi \frac{k}{k_\alpha^2} \delta(\Omega - \Omega_0) J_0(kr) \left(1 - \frac{k^2}{k_\alpha^2}\right)^{-\frac{1}{2}} dk. \quad (10)$$

If in Eq. 8 we put  $\Omega = 2\pi(1 - \cos\theta)$ , then

$$d\Omega = 2\pi \sin\theta d\theta = 2\pi \sin\theta (d\theta/dk) dk$$

and

$$dk = (d\Omega/2\pi \sin\theta) k_\alpha \cos\theta.$$

The correlation function becomes

$$\int_0^{k_\alpha} \delta(\Omega - \Omega_0) J_0(kr) d\Omega = J_0(k_\alpha r \sin\theta_0). \quad (11)$$

### C. Two-Dimensional Reverberant Field

The equation corresponding to Eq. 4 may be written

$$p_\theta(\theta) d\theta = (1/2\pi) d\theta \quad (12)$$

and as

$$p_\theta(\theta) d\theta = \frac{1}{2} f_k(k) dk.$$

We see that

$$2\pi f_k(k) = (2/k_\alpha) [(1 - k^2)/k_\alpha^2]^{-\frac{1}{2}}. \quad (13)$$

The inverse transform may thus be written as

$$\frac{1}{\pi} \int_{-k_\alpha}^{k_\alpha} \left(1 - \frac{k^2}{k_\alpha^2}\right)^{-\frac{1}{2}} k_\alpha \exp(jkr) dk = J_0(k_\alpha r). \quad (14)$$

### D. Plane-Wave Incidence

In this case,

$$p(\theta) = \delta(\theta - \theta_0)$$

and

$$f_k(k) = \delta(\theta - \theta_0) \frac{2\pi}{k_\alpha} \left(1 - \frac{k^2}{k_\alpha^2}\right)^{-\frac{1}{2}}. \quad (15)$$

Again the inverse transform is given by

$$\int_{-k_\alpha}^{k_\alpha} \delta(\theta - \theta_0) k_\alpha \left(1 - \frac{k^2}{k_\alpha^2}\right)^{-\frac{1}{2}} \exp(jkr) dk = \exp(jk_\alpha r \sin\theta_0). \quad (16)$$

## II. SUMMARY

Although none of the results for the correlation function for different kinds of spatial distribution is new, the method of analysis has some novelty. When we wish to consider transmission through plates, for example, there is some advantage in expressing the plate impedance in terms of  $k$  parameters. Then any deviation from the ideally reverberant incident field may be adjusted by modifying the incident  $k$  spectra.

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<sup>1</sup> R. K. Cook *et al.*, J. Acoust. Soc. Amer. 27, 1072 (1955).

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