Correlation of reverberant sound fields using a k- transform technique

Donato, R. J.

Publisher's version / Version de l'éditeur:

Journal of the Acoustical Society of America, 51, 5(pt. 2), pp. 1764-1765, 1972-01-01

NATIONAL RESEARCH COUNCIL OF CANADA
CONSEIL NATIONAL DE RECHERCHES DU CANADA

CORRELATION OF REVERBERANT SOUND FIELDS USING A k-TRANSFORM TECHNIQUE

by

R. J. DONATO

Reprinted from
JOURNAL OF THE ACOUSTICAL SOCIETY OF AMERICA
Vol. 51, No. 5, Part 2, May 1972
pp. 1764–1765

Research Paper No. 505
of the
Division of Building Research

OTTAWA
January 1972

Price 10 cents

NRCC 12337

LA CORRELATION DES CHAMPS SONORES REVERBERANTS PAR UNE METHODE DE "K" TRANSFORMES

SOMMAIRE

L'auteur décrit une méthode de transformes servant à calculer la corrélation spatiale de divers genres de champs acoustiques. On insiste sur le fait que l'emploi d'une méthode de "k" nombres pour déterminer un champ sonore équivaut à la synthèse d'un champ sonore en tant que sommation d'une série d'ondes planes.

Reprinted from: The Journal of the Acoustical Society of America

11.2, 11.7; 2.7; 13.6

Received 27 September 1971

Correlation of Reverberant Sound Fields Using a *k*-Transform Technique

R. J. DONATO

Building Physics Section, Division of Building Research, National Research Council of Canada, Ottawa, Canada

A transform method is described for calculating the spatial correlation for various kinds of acoustic fields. The method emphasizes the equivalence between using a k-number approach for specifying a sound field and the synthesis of a sound field as a summation of a series of plane waves.

The usual way of determining the spatial correlation of a three- or two-dimensional reverberant sound field is to decompose the original field into a system of plane waves, calculate the effect for a small bundle of such waves, and then integrate for the whole set.[1]

Specifically, in the three-dimensional case, the correlation function $\rho(d,\tau)$ for spatial separation d and time difference τ is given by

$$\rho(d,\tau)=(4\pi)^{-1}\int_0^{2\pi}\int_0^{\pi}\rho\left(\tau+\frac{d\cos\theta}{c}\right)\sin\theta d\theta d\varphi, \quad (1)$$

where c is the propagation velocity, and θ, φ are the polar and azimuth angles in spherical coordinates; the factor 4π ensures normalization. When we are considering sine waves we may write

$$\rho\left(\tau+\frac{d\cos\theta}{c}\right)=\cos\omega\left(\tau+\frac{d\cos\theta}{c}\right), \quad (2)$$

and we may write Eq. 1 for $\tau=0$ as

$$\rho(d,0)=(4\pi)^{-1}\left|\frac{\sin(\omega d\cos\theta/c)}{\omega d/c}\right|_{\pi}^{0}2\pi$$

$$=\frac{\sin(\omega d/c)}{\omega d/c}. \quad (3)$$

I. TRANSFORM METHOD

Sometimes when dealing with sound transmission problems it is more convenient to deal with the wavenumber parameter k than the angle of incidence θ. The following examples emphasize the equivalence of the two approaches. The first example evaluates the three-dimensional reverberant field case given above.

A. Three-Dimensional Reverberant Field

Let $p_\Omega(\Omega)\cdot d\Omega$ represent the distribution of energy within solid angles Ω and $\Omega+d\Omega$. For an azimuthly symmetric system, we may write the distribution as $p_\theta(\theta)$ where, of course,

$$p_\Omega(\Omega)d\Omega=p_\theta(\theta)d\theta.$$

If we consider a three-dimensional reverberant space, then

$$p_\theta(\theta)=(4\pi)^{-1}2\pi\sin\theta=\tfrac{1}{2}\sin\theta. \quad (4)$$

For such an axially symmetric system we may write

$$kf_k(k)dk=2(\tfrac{1}{2}\sin\theta d\theta), \quad (5)$$

where we have now expressed the distribution in terms of $f_k(k)$. Equation 5 equates the energy for a band dk in the k plane to that in band $d\theta$ in the θ plane.

Writing $k=k_\alpha\sin\theta$, where k_α is the wave propagation number, then

$$kf_k(k)=\sin\theta\frac{d\theta}{dk}=\frac{k}{k_\alpha^2}\left(1-\frac{k^2}{k_\alpha^2}\right)^{-\frac{1}{2}}. \quad (6)$$

Equation 6 is a measure of the power k spectrum of the reverberant sound field. By taking the inverse transform—in this case Hankel transform as we have assumed axial symmetry—and using the Wiener–Khintchine theorem, we may derive the correlation function between two points separated in space. Thus we may write for the spatial correlation coefficient

$$\int_0^{k_\alpha}\frac{k}{k_\alpha^2}J_0(kr)\left(1-\frac{k}{k_\alpha^2}\right)^{-\frac{1}{2}}dk=\frac{\sin k_\alpha r}{k_\alpha r}. \quad (7)$$

The upper limit of k must be k_α to correspond to the upper limit of $\theta=\pi/2$.

An interesting sidelight is obtained from Eq. 7. The right-hand side is such that one would expect a constant bounded k spectrum if the correct variables were chosen. If one chooses k_z, the wavenumber perpendicular to the surface being considered, then

$$f(k_z)dk_z=2\pi\sin\theta d\theta$$

and

$$f(k_z)=\text{constant}(k_\alpha)^{-1}.$$

Then by a suitable choice of axes we may simplify the derivation of Eq. 7.

B. Incident Field from an Annulus

For this case, we shall take $p_\Omega(\Omega)=\delta(\Omega-\Omega_0)$; i.e., the sound field is restricted to a narrow band of solid angles around Ω_0, where $\delta(\Omega-\Omega_0)$ is the impulse function and

$$\int_{-\infty}^{\infty}\delta(\Omega-\Omega_0)d\Omega=1.$$

We may then write

$$p_\theta(\theta)=2\pi\sin\theta\,\delta(\Omega-\Omega_0). \tag{8}$$

Proceeding as before,

$$kf_k(k)=2\pi(k/k_\alpha^2)\delta(\Omega-\Omega_0)(1-k^2/k_\alpha^2)^{-\frac{1}{2}}. \tag{9}$$

Similarly, the inverse transform will be given by

$$\int_0^{k_\alpha}2\pi\frac{k}{k_\alpha^2}\delta(\Omega-\Omega_0)J_0(kr)\left(1-\frac{k^2}{k_\alpha^2}\right)^{-\frac{1}{2}}dk. \tag{10}$$

If in Eq. 8 we put $\Omega=2\pi(1-\cos\theta)$, then

$$d\Omega=2\pi\sin\theta d\theta=2\pi\sin\theta(d\theta/dk)dk$$

and

$$dk=(d\Omega/2\pi\sin\theta)k_\alpha\cos\theta.$$

The correlation function becomes

$$\int_0^{k_\alpha}\delta(\Omega-\Omega_0)J_0(kr)d\Omega=J_0(k_\alpha r\sin\theta_0). \tag{11}$$

C. Two-Dimensional Reverberant Field

The equation corresponding to Eq. 4 may be written

$$p_\theta(\theta)d\theta=(1/2\pi)d\theta \tag{12}$$

and as

$$p_\theta(\theta)d\theta=\tfrac{1}{2}f_k(k)dk.$$

We see that

$$2\pi f_k(k)=(2/k_\alpha)[(1-k^2)/k_\alpha^2]^{-\frac{1}{2}}. \tag{13}$$

The inverse transform may thus be written as

$$\frac{1}{\pi}\int_{-k_\alpha}^{k_\alpha}\left(1-\frac{k^2}{k_\alpha^2}\right)^{-\frac{1}{2}}k_\alpha\exp(jkr)dk=J_0(k_\alpha r). \tag{14}$$

D. Plane-Wave Incidence

In this case,

$$p(\theta)=\delta(\theta-\theta_0)$$

and

$$f_k(k)=\delta(\theta-\theta_0)\frac{2\pi}{k_\alpha}\left(1-\frac{k^2}{k_\alpha^2}\right)^{-\frac{1}{2}}. \tag{15}$$

Again the inverse transform is given by

$$\int_{-k_\alpha}^{k_\alpha}\delta(\theta-\theta_0)k_\alpha\left(1-\frac{k^2}{k_\alpha^2}\right)^{-\frac{1}{2}}\exp(jkr)dk =\exp(jk_\alpha r\sin\theta_0). \tag{16}$$

II. SUMMARY

Although none of the results for the correlation function for different kinds of spatial distribution is new, the method of analysis has some novelty. When we wish to consider transmission through plates, for example, there is some advantage in expressing the plate impedance in terms of k parameters. Then any deviation from the ideally reverberant incident field may be adjusted by modifying the incident k spectra.

ACKNOWLEDGMENTS

This paper is a contribution from the Division of Building Research, National Research Council of Canada, and is published with the approval of the Director of the Division.

[1] R. K. Cook *et al.*, J. Acoust. Soc. Amer. **27**, 1072 (1955).

Publications of the Division may be obtained by mailing the appropriate remittance (a Bank, Express, or Post Office Money Order, or a cheque, made payable to the Receiver General of Canada, credit NRC) to the National Research Council of Canada, Ottawa. KIA0R6. Stamps are not acceptable.

A list of all publications of the Division is available and may be obtained from the Publications Section, Division of Building Research, National Research Council of Canada, Ottawa. KIA0R6.